An Understanding of Induction Motor Performance through Electrical Signature Analysis Part 1

Howard W Penrose, Ph.D., CMRP, CEM, CMVP
President, MotorDoc LLC
info@motordoc.com

Abstract: Electrical Signature Analysis (ESA) is a powerful prognostic and performance tool for analyzing electric machinery through the measurement of the effects on the magnetic field as observed in voltage and current. This white paper explores how various induction motor and load conditions affect the magnetic field and spectral characteristics of measured values. We will delve into the mathematical models for radial and torsional conditions, the impact of driven loads, and the influence of power quality on the motor's performance.

INTRODUCTION

ESA is a powerful diagnostic tool for detecting and diagnosing defects and performance of electric motors by analyzing voltage and current waveforms. This paper explores how various conditions affect the magnetic field and spectral characteristics of induction motors and driven equipment, including power quality impact. We will delve into the mathematical models for radial and torsional defects, the impact of drive loads, and the influence of power quality on the motor's performance. Additionally, we will examine how the severity of defects is represented in the dB spectrum under different load conditions.

For the purposes of the figures used in this paper we will assume an 1800 RPM motor that operates at 25%, 50%, 75% and 100% load with current at 100 Amps. We will not assume voltage for the purpose of analysis for this paper, but will include it and torsional/kilowatt analysis in Part 2.

MAGNETIC FIELD EFFECTS

The transducer for ESA is the magnetic field between the rotor and stator. The variations affect the field radially and torsionally, and, from a practical standpoint, normally a combination of both.

RADIAL DEFECTS

Common radial defects include electric machinery conditions including eccentricity, rotor deformation or defects, bearing wear, and stator conditions such as coil movement or missing/damaged wedges. These include:

- Eccentricity: this can include static (constant displacement, such as rotor off center) or dynamic (magnetic rotation in the airgap).

- Rotor Deformation or Damage: this can include fractured rotor bars, casting voids, non-uniform surfaces, bent shaft, loose rotor on shaft, and similar conditions. This can be in conjunction with eccentricity.
- Stator: missing wedges, coil movement including in-slot or overly long coil ends, stator core movement, winding degradation, partial discharge, slot sparking, and other conditions.
- Bearing Wear: inner and outer race, ball and cage, lubrication, electrical discharge, sleeve bearing conditions (in conjunction with eccentricity), thrust, misalignment, other conditions that involve increased losses.

These radial defects cause non-uniformities in the air gap leading to variations in magnetic reluctance. The air gap flux density $B_g(\theta)$ can be modeled as shown in Eq. 1.

$$B_g(\theta) = B_O[1 + \sum_{n=1}^{\infty} \epsilon_n \cos(n\theta + \emptyset_n)] \qquad \text{Eq. 1}$$

Where:

- B_O is the nominal air gap flux density.
- ϵ_n represents the magniture of the nth harmonic due to defects.
- θ is the angular position.
- $\emptyset_n$ is the phase angle of the nth harmonic

The non-uniform magnetic field induces non-ideal back electromotive force (EMF) and distorts the current waveform. This can be represented as Eq. 2.

$$e(t) = E_0 \sin(\omega t + \emptyset) + \sum_{n=1}^{\infty} E_n \sin(n\omega t + \emptyset_n) \qquad \text{Eq. 2}$$

Where:

- E_0 is the fundamental EMF.
- E_n represents the nth harmonic of the EMF.

The distorted EMF leads to a distorted current waveform, *I(t)*, given by Eq. 3.

$$I(t) = \frac{E(t)}{Z(\omega)} \qquad \text{Eq. 3}$$

Where $Z(\omega)$ is the impedance of the motor.

TORSIONAL EFFECTS

Torsional effects are primarily the result of forces on the shaft from the driven load. This can be anything from defects in the coupling, gears, bearings, sheaves and belts, and similar, or misalignment between the electric machine and the load, or other driven equipment. Torsional defects cause periodic variations in the position of the rotor, leading to time-varying air gap lengths. The time-varying air gap length $g(t)$ can be modeled as shown in Eq. 4.

$$g(t) = g_0[1 + \gamma \sin(\omega_m t + \emptyset_m)] \qquad \text{Eq. 4}$$

Where:

- g_0 is the nominal air gap length.
- γ is the amplitude of the variation due to torsional defects.
- ω_m is the mechanical angular frequency.
- $\emptyset_m$ is the phase angle of the variation.

The time-varying air gap affects the magnetic linkage, which impacts the magnetic field and current waveform, and can be expressed as shown in Eq. 5.

$$\Phi(t) = \Phi_0\left[1 + \sum_{n=1}^{\infty} \gamma_n \sin\left(n\omega_m t + \emptyset_{m,n}\right)\right] \qquad \text{Eq. 5}$$

Where:

- Φ_0 is the nominal flux linkage.
- γ_n represents the nth harmonic of the flux variation.

This induces harmonics in the back EMF and affects the current waveform similarly to radial defects, leading to the same formula as shown in Eq. 2 and Eq. 3.

COMBINED EFFECTS

In a practical scenario, both radial and torsional defects usually coexist, and their combined effects on the magnetic field and current waveform will be more complex. $\triangle g(\theta, t)$ is the overall airgap variation as modeled in Eq. 6.

$$\triangle g(\theta, t) = \triangle g_{radial}(\theta) + \triangle g_{torsional}(\theta t) \qquad \text{Eq. 6}$$

The combined magnetic field $B_g(\theta, t)$ then becomes as shown in Eq. 7.

$$B_g(\theta, t) = B_O\left[1 + \sum_{n=1}^{\infty} \epsilon_n \cos(n\theta + \emptyset_n) + \sum_{m=1}^{\infty} \gamma_n \sin\left(m\omega_m t + \emptyset_{m,m}\right)\right] \qquad \text{Eq. 7}$$

The combined EMF and current waveform ends up including harmonics from both radial and torsional defects, leading to Eq. 8.

$$e(t) = E_0 \sin(\omega t + \emptyset) + \sum_{n=1}^{\infty} E_n \sin(n\omega t + \emptyset_n) + \sum_{m=1}^{\infty} E_m \sin\left(m\omega_m t + \emptyset_{m,m}\right) \qquad \text{Eq. 8}$$

Defects of any severity in an electric motor, both radially for internal and torsional for driven equipment, impact the magnetic air gap, leading to variations in the magnetic field. These variations induce harmonics associated with the frequencies related to defective components in the back EMF, which distorts the waveform. Accurate modeling and analysis of these effects are excellent for diagnosing motor health and ensuring efficient operation.

IMPACT OF DEFECTS ON SPECTRA

The values of the magnetic field airgaps are relatively small. To understand the impact and then show a relative difference using the log scale (decibels, dB) we will discuss linear spectra (absolute values) and dB spectral analysis. The purpose of the conversion to dB is the ability to provide a value that remains constant regardless of load such that the values are relative to the peak voltage or current. For the purpose of this white paper, we will review bearing defects and rotor bar fractures.

Bearing defects typically introduce characteristic frequencies related to the defect type. These frequencies are calculated based upon the gearing geometry and rotational speed. In most cases the 1Hz speed values for the bearing components are published. Common ball and roller bearing defect frequencies include:

- Ball Pass Frequency Outer Race (BPFO): f_{BPFO}
- Ball Pass Frequency Inner Race (BPFI): f_{BPFI}
- Fundamental Train Frequency (FTF, also referred to as the cage frequency): f_{FTF}
- Ball Spin Frequency (BSF): f_{BSF}

The current signal *I(t)* with bearing defects can be modeled as shown in Eq. 9.

$$I_{bearing}(t) = I_0 \sin(\omega t) + \sum_k A_k \sin(2\pi f_k t + \phi_k) \qquad \text{Eq. 9}$$

Where f_k are the characteristic bearing defect frequencies and A_k are their amplitudes.

Rotor bar fractures introduce sidebands around the fundamental frequency. These sidebands are observed at twice the slip frequency f_s. The current signature *I(t)* with rotor bar fractures can be modeled as shown in Eq. 10.

$$I_{rotor}(t) = I_0 \sin(\omega t) + \sum_n B_n \sin(\omega t \pm 2\pi n f_s t + \phi_n) \qquad \text{Eq. 10}$$

Where f_s is the slip frequency and B_n are the amplitudes of the sidebands. Combined, the bearing and rotor defects from the fields would appear as shown in Eq. 11.

$$I_{combined}(t) = I_0 \sin(\omega t) + \sum_k A_k \sin(2\pi f_k t + \phi_k) + \sum_n B_n \sin(\omega t \pm 2\pi n f_s t + \phi_n) \quad \text{Eq. 11}$$

LINEAR SPECTRUM REPRESENTATION

The linear spectrum *I(f)* of the current signature can be obtained by taking the Fourier Transform of *I(t)* as shown in Eq. 12.

$$I(f) = \mathcal{F}\{I(t)\} \qquad \text{Eq. 12}$$

This will show peaks at:

- The fundamental frequency $\omega/2\pi$.

- Bearing defect frequencies f_k.
- Sidebands around the fundamental and harmonics due to rotor bar fractures.

DECIBEL SPECTRA

To convert the linear spectra to dB spectra we refer to Eq. 13.

$$I_{dB}(f) = 20\log_{10}\left(\frac{I(f)}{I_{max}}\right) \qquad \text{Eq. 13}$$

Where I_{max} is the peak current amplitude, which is used for the reference level (0 dB).

The presence of defects such as bearing defects and rotor bar fractures introduces specific frequencies and sidebands into the current spectrum. By analyzing the linear and dB spectra the characteristic frequencies associated with defects can be identified. However, can the power quality signatures also impact the ability to read the signatures and identify defects?

POWER QUALITY IMPACT ON MAGNETIC FIELD DATA

Power quality issues such as voltage sags, swells, harmonics and unbalanced voltage affect the current waveform, which in turn affects the magnetic field. How this is viewed in the spectra can determine if it can be separated from defect frequencies. For the purpose of this white paper we will evaluate voltage harmonic impacts.

Voltage harmonics introduce corresponding current harmonics as shown in Eq. 14.

$$V(t) = V_0\sin(\omega t) + \sum_{k=2}^{\infty} V_k \sin(k\omega t + \phi_k) \qquad \text{Eq. 14}$$

Which results in a distorted current as outline in Eq. 15.

$$I(t) = I_0\sin(\omega t) + \sum_{k=2}^{\infty} I_k \sin(k\omega t + \phi_k) \qquad \text{Eq. 15}$$

The distorted current affects the magnetic field as shown in Eq. 16.

$$B(t) = B_0\sin(\omega t) + \sum_{k=2}^{\infty} B_k \sin(k\omega t + \phi_k) \qquad \text{Eq. 16}$$

Overall, radial and torsional defects introduce specific harmonic variations in the magnetic field as do the torsional effects related to constant and variable driven loads. These have specific frequencies determined by their multipliers associated with running speed, which is generally not synchronous in the types of machines being evaluated in this paper. Power quality conditions such as harmonic distortion are direct multipliers of line frequency and are directly related to the magnetic field disturbance, which are not associated with most fault frequencies.

DEFECT SEVERITY AND LOAD EFFECTS IN SPECTRA

As the mathematical model and proofs associated with the ability of ESA to identify defects, the severity of the associated defects becomes important. If we compare the relationship of linear versus log-log spectrum (dB), which method provides the more accurate and trend-able level regardless of load?

The dB value in the spectral analysis can be used to indicate the severity of defects. The larger the amplitude of the harmonics or sidebands in the dB spectra, the more severe the defect. We will model this relationship mathematically and show how it is affected by load variations. We will then provide examples of exaggerated progressive defects: bearings, misalignment, and rotor fractures.

RELATIONSHIP BETWEEN dB VALUE AND DEFECT SEVERITY

The dB values for a frequency component was explored in Eq. 13 in which the severity of the defect can be quantified by the increase in the amplitude of its characteristic frequency components in the spectrum. The distortion of the defect causes the defect frequency current to increase should the fundamental frequency remain constant. However, what happens if the load changes during the evaluation? Is there a requirement that the load must be the same in order to compare results in linear versus log spectrum analysis?

Let's first start with the bearing and a progression. We'll assume the following progression and Eq. 17 and 18:

$$I_{bearing}(t) = I_0 \sin(\omega t) + \sum_k A_k \sin(2\pi f_k t + \phi_k) \qquad \text{Eq. 17}$$

$$I_{dB}(f_k) = 20 \log_{10}\left(\frac{A_k}{I_0}\right) \qquad \text{Eq. 18}$$

- Initial stage: $A_{BPFO} = 0.1\, I_0$
- Intermediate stage: $A_{BPFO} = 0.2\, I_0$
- Severe stage: $A_{BPFO} = 0.4\, I_0$

The dB values are, as a result (for purposes of the formula, and exaggeration, the peak current is 1 Amp:

- Initial stage: $I_{dB}(f_{BPFO}) = 20 \log_{10}\left(\frac{0.1 I_0}{I_0}\right) = -20\ dB$
- Intermediate stage: $I_{dB}(f_{BPFO}) = 20 \log_{10}\left(\frac{0.2 I_0}{I_0}\right) = -14\ dB$
- Severe stage: $I_{dB}(f_{BPFO}) = 20 \log_{10}\left(\frac{0.4 I_0}{I_0}\right) = -8\ dB$

If we then double the load to 2 Amps and the associated defect to double values, then the following would be found:

- Initial stage: $I_{dB}(f_{BPFO}) = 20\log_{10}\left(\frac{0.2I_0}{I_0}\right) = -20\ dB$
- Intermediate stage: $I_{dB}(f_{BPFO}) = 20\log_{10}\left(\frac{0.4I_0}{I_0}\right) = -13.98\ dB$
- Severe stage: $I_{dB}(f_{BPFO}) = 20\log_{10}\left(\frac{0.8I_0}{I_0}\right) = -7.96\ dB$

As a result, the effective dB values remain the same. Based on the theoretical scenario these values will be roughly the same, with only small differences, while the current values double.

EXAMPLE REVIEW OF A MOTOR

For the purposes of graphical demonstration we will include the defects identified in this paper applied to a 100 Amp, 1800 RPM motor operating at 1795 RPM, 1790 RPM, 1785 RPM and 1780 RPM. The loads are 25%, 50%, 75% and 100% for the evaluation.

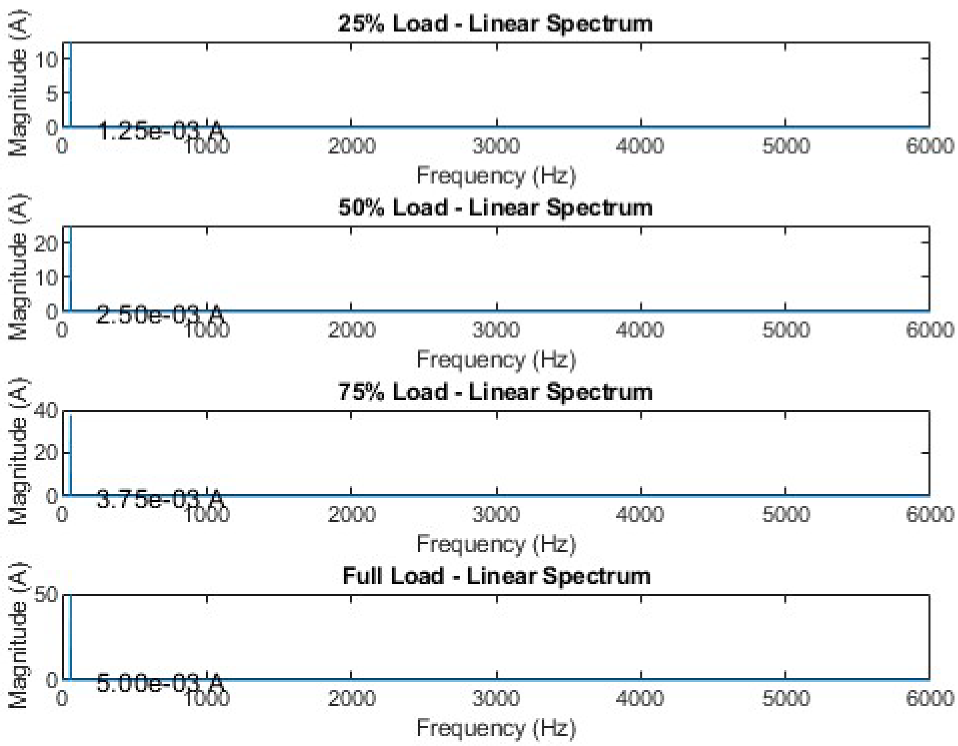

Figure 1: Linear spectrum of bearing defects.

BPFO peaks in the linear scale (Figure 1):

- 25% load: 0.00125 Amps
- 50% load: 0.00250 Amps
- 75% load: 0.00375 Amps
- 100% load: 0.00500 Amps

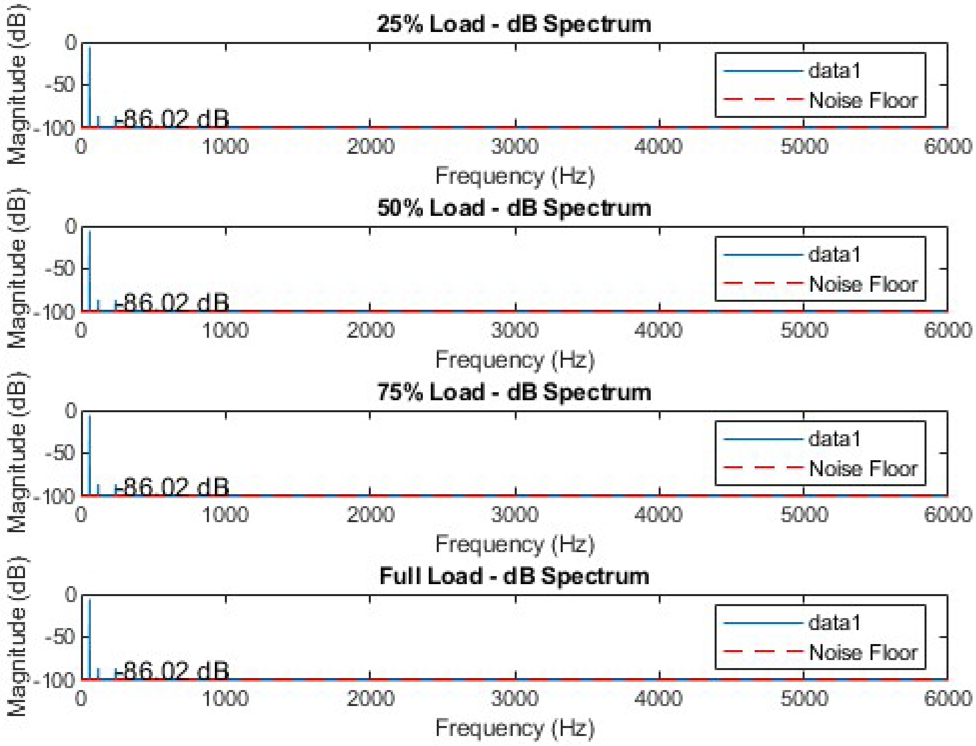

Figure 2: Linear converted to dB for the bearing defect at different loads.

BPFO peaks in the dB scale (Figure 2):

- 25% load: -86.02 dB
- 50% load: -86.02 dB
- 75% load: -86.02 dB
- 100% load: -86.02 dB

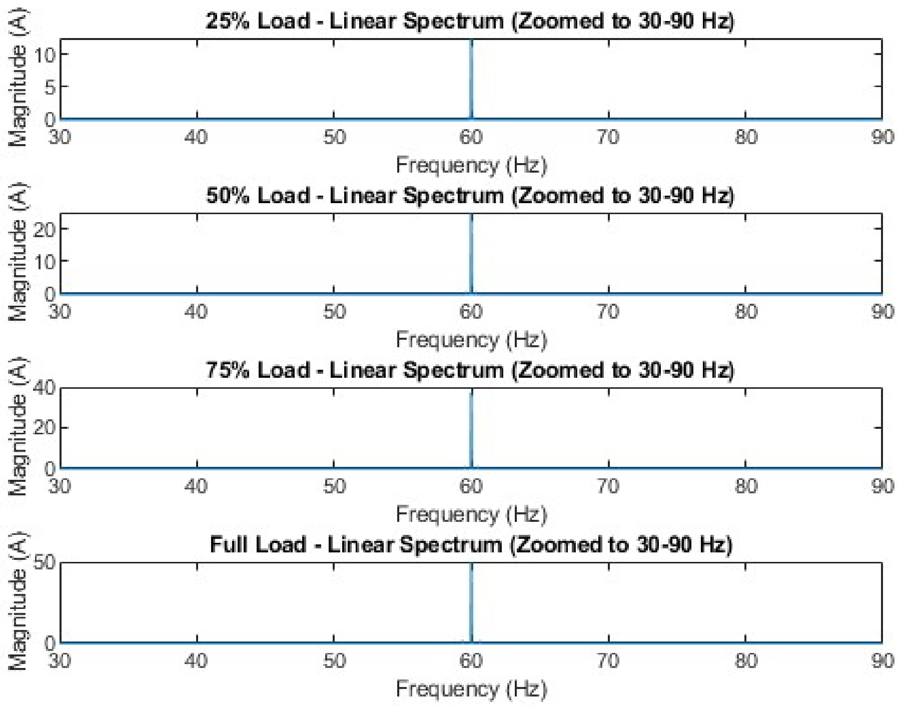

Figure 3: Linear spectrum of PPF sidebands at the different loads and resulting in values that are not clearly visible.

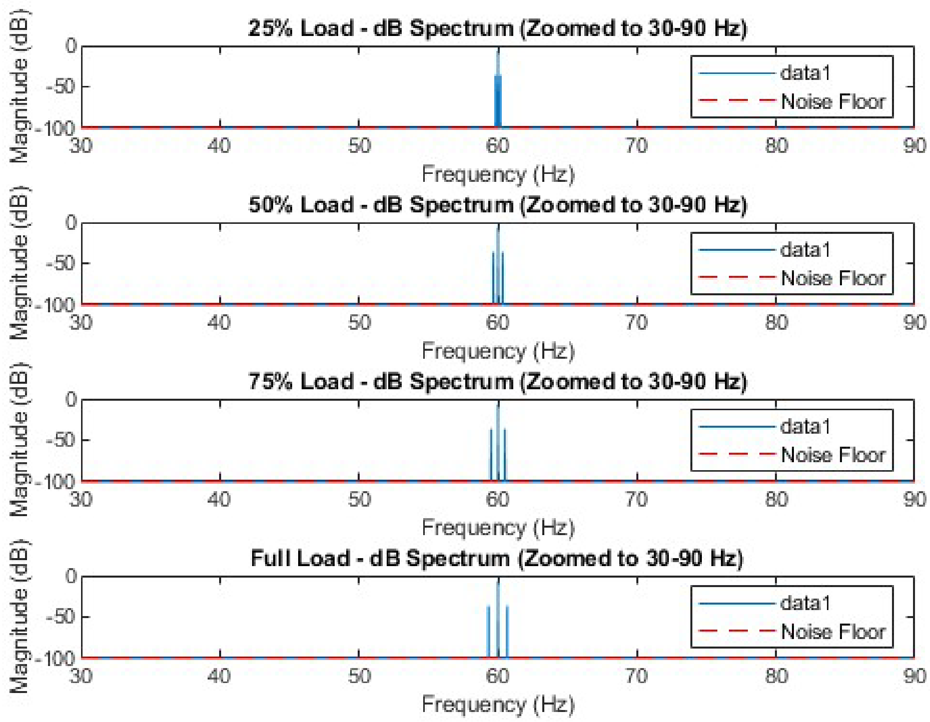

Figure 4: Peaks are the same value but different slip frequencies.

CONCLUSION

Electrical Signature Analysis (ESA) proves to be an effective tool for assessing the performance and health of induction motors. By analyzing the impact of radial and torsional defects, along with the power quality issues, on the motor's magnetic field and spectral characteristics, ESA provides a comprehensive method for identifying and quantifying defects. The use of decibel spectra enables consistent trending of defect severity, independent of load variations, ensuring accurate, reliable and trend-able diagnostics. This first part of a multi-part study underscores the importance of rules-based ESA in maintaining efficient motor operation and preventing unforeseen failures through early detection and monitoring.

